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CONVOLUTION THEOREM OF MELLIN-WAVELET TRANSFORM

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ABSTRACT:

The Mellin transform is an integral transform that may regarded as the multiplicative version of the two sided Laplace transform. Mellin transform is basic tool for analyzing the behaviour of many in Mathematics and mathematical physics. The Mellin transform is widely used in computer science for the analysis of algorithms because of its scale invariance property. Mellin transform has many applications such as navigation, radar system, in finding the stress distribution in an infinite wedge, also in digital audio effects. Mellin transform, a kind of nonlinear transformation, is widely used for its scale invariance property. So it has special importance in scale representation of signal. The Wavelet transform has been shown to be a successful tool for dealing with transient signals, data compression, sound analysis, representation of the human retina. The Wavelet transform is done similar like to Short Term Fourier Transform (STFT) analysis. In this paper convolution theorem of Mellin-Wavelet transform is proved.

Keywords :- Signal processing, Mellin transform, Wavelet transform, Testing function space, Mellin-Wavelet transform.

INTRODUCTION :

The Mellin integral transform is an important tool in Mathematics. Mathematics is everywhere in every phenomenon, technology, observation, experiment etc. The Mellin transform is an integral transform named after the finnish mathematician Hjalmar Mellin (1854-1933). He developed applications to the solution of hypergeometric differential equations and to the derivation of asymptotic expansions. The Mellin transform used in place of Fourier transform when scale invariance is more relevant than shift invariance [4]. The Mellin transform is used in signal processing as a tool to investigate scale invariance and it gives a transform space image that is invariant to translation, rotation and scale [3]. Besides its use in Mathematics, Mellin transform has been applied in many different areas of Physics and Engineering. We use the Mellin integral transform to derive different properties Statistics and probability densities of single continuous random variable [2,7].

Wavelet transform is emerged as the most effective technique for signal processing and image analysis as an alternative to Fourier analysis especially when the signals are random, comparised of fluctuations of different scales and where the very short and very long waves are present in the same signal. The signals are either deterministic or random. The deterministic signals are usually by mathematical functions. A Wavelet is a wave-like oscillation that is localized in time. Wavelets are mathematical tool, they can be used to extract information from many different kinds of data, including audio signals and images.The Wavelet transform is of interest for the analysis of non-stationary signals because it provides an alternative to classical linear time-frequency representations with better time and frequency localization properties [14]. The main objective of Wavelet transform is to define the powerful wavelet basis function and find efficient methods for their computations. On the other

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hand, the role of the other Wavelet transformation is to remove the non-stationary properties of the involved signals; consequently, the conventional estimation algorithms for stationary signal processing can be employed in each scale of the Wavelet domain [5]. At high frequencies, the Wavelet transform gives good time resolution. While at low frequencies, the Wavelet transform gives good frequency resolution and poor time resolution.

Convolution is an operation involving two functions that turns out to be rather useful in many applications. We have two reasons for introducing it here. First of all, Convolution will give us a way to deal with inverse transforms of fairly arbitrary products of functions . Secondly, it will be a major element in relatively formulas for solving a number of differential equations. The main aim of this paper is to present the convolution theorem for Mellin-Wavelet transform.

Mellin –Wavelet Transform

The Conventional Mellin –Wavelet transform is defined as

$$MW_{\Psi}\{f(t,x)\} = MW_{\Psi}(p,a,b)$$
$$= \int_0^{\infty}\int_{-\infty}^{\infty} f(t,x)\, K(t,x,p,a,b)\; dt\, dx$$

where $K(t,x,p,a,b) = \frac{1}{|a|^{\frac{1}{2}}} t^{p-1} e^{i\pi\left(\frac{x-b}{a}\right)^2}$

Distributional Generalized Mellin- Wavelet Transform

For $f(t,x) \in MW^{*}_{a,b,\Psi,p}$

where $MW^{*}_{a,b,\Psi,p}$ is the dual space of $MW_{a,b,\Psi,p}$ and $m < Re\, p < n,\ b \in R,\ a \neq 0$

the distributional Mellin-Wavelet transform is defined as

$$MW_{\Psi}\{f(t,x)\} = MW_{\Psi}(p,a,b) = \langle f(t,x), K(t,x,p,a,b)\rangle$$

where $K(t,x,p,a,b) = \frac{1}{|a|^{\frac{1}{2}}} t^{p-1} e^{i\pi\left(\frac{x-b}{a}\right)^2} \in MW_{a,b,\Psi,p}$

The Test Function

An infinitely differentiable complex valued function $\Phi(t,x)$ on R belongs to $E(R)$ if for each compact set $K \subset S_{\Psi}, p \subset S_{\Psi}$

where $S_{g,h} = \{t,x: t,x \in R, |t| \le g, |x| \le h, g > 0, h > 0\}$

$$\gamma_{l,k,p}\phi(t,x) = \sup_{I}\left|\xi_{m.n}(t)t^{q+1}D_t^q D_x^k \phi(t,x)\right| < \infty$$

where

q, k=0,1,2,3………

and

$$\xi_{m,n} = \begin{cases} t^{-m} & ;0 < t \le 1 \\ t^{-n} & ;1 < t < \infty \end{cases}$$

Thus $E(R)$ will denote the space of all $\phi \in E(R)$ with compact support in S_{Ψ}.

Moreover, we say that f is a Mellin-Wavelet transformable if it is a member of E.

Convolution Theorem:

Statement:

Let $h(t,x) = (f * g)(t,x)$ and F, G and H denote the Mellin-Wavelet transform of f , g and h respectively. Then $H(p,a,b) = C^2 F(p,a,b)\, G(p,a,b)$

Proof: From the definition of Mellin- Wavelet Transform

$$H(p,a,b) = \int_0^{\infty}\int_{-\infty}^{\infty} f(t,x)\, K(t,x,p,a,b)\, h(t,x)\; dt\, dx$$

$$= \int_0^{\infty}\int_{-\infty}^{\infty} ct^{p-1}e^{iC_1(x-b)^2}\, h(t,x)\; dt\, dx$$

Where $C = \frac{1}{|a|^{\frac{1}{2}}}$ and $C_1 = \frac{\pi}{a^2}$

$$= \int_0^{\infty}\int_{-\infty}^{\infty} Ct^{p-1}e^{iC_1(x-b)^2}\left[Ct^{-(p-1)}e^{-iC_1(x-b)^2}(\tilde{f} * \tilde{g})(t,x)\right] dt\, dx$$

$$= C^2\int_0^{\infty}\int_{-\infty}^{\infty} t^{p-1}t^{-(p-1)}e^{iC_1(x-b)^2}e^{-iC_1(x-b)^2}\left[\int_0^{\infty}\int_{-\infty}^{\infty}\tilde{f}(u,v)\tilde{g}(t-u,x-v)\, du\, dv\right] dt\, dx$$

$$= C^2\int_0^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_{-\infty}^{\infty} f(u,v)\, C\, u^{p-1}e^{iC_1(v-b)^2}\, g(t-u,x-v)\, C\,(t-u)^{(p-1)}e^{iC_1(x-v-b)^2}\; du\, dv\, dt\, dx$$

$$= C^2\int_0^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_{-\infty}^{\infty} u^{p-1}e^{iC_1(v-b)^2}f(u,v)(t-u)^{(p-1)}\, g(t-u,x-v)e^{iC_1(x-v-b)^2}\; du\, dv\, dt\, dx$$

Put $t-u = y$ And $x-v = z$

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$\Rightarrow \quad dt = dy \qquad\qquad \Rightarrow \quad dx = dz$

$$\therefore H(p,a,b) = C^4 \int_0^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_{-\infty}^{\infty} u^{p-1} e^{iC_1(v-b)^2} y^{(p-1)} e^{iC_1(z-b)^2} f(u,v) g(y,z) \; du\, dv\, dy\, dz$$

$$= C^2\left(C\int_0^{\infty}\int_{-\infty}^{\infty} u^{p-1}\, e^{iC_1(v-b)^2} f(u,v)\, dudv\right)\left(C\int_0^{\infty}\int_{-\infty}^{\infty} y^{(p-1)} e^{iC_1(z-b)^2} \; g(y,z) dy\, dz\right)$$

$$= C^2\, F(p,a,b)\, G(p,a,b)$$

$$\therefore H(p,a,b) = C^2\, F(p,a,b)\, G(p,a,b)$$

Where $\quad C = \frac{1}{|a|^{\frac{1}{2}}} \quad$ and $C_1 = \frac{\pi}{a^2}$

CONCLUSION :

In the present work we mainly focused on the Convolution Theorem of Mellin-Wavelet transform.

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